

# Complementation of Coalgebra Automata

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# Goal of the Talk

## Theorem

*The class of languages recognisable by  $\mathcal{T}$ -coalgebra automata is closed under taking complements.*

# Outline

1. One Step Complementation Lemma
  - ▶ Moss' Modality
  - ▶ Boolean Dual of Moss' Modality
2. Game Bisimulation
  - ▶ Parity Graph Games
  - ▶ Basic Sets and Local Games
  - ▶ Powers and Game Normalisation
  - ▶ Game Bisimulation
3. Complementation Lemma for Coalgebra Automata
  - ▶ Coalgebra Automata
  - ▶ Complementation of Transalternating Automata
  - ▶ Equivalence of Transalternating, Semi-transalternating and Alternating Automata

# Category-Theoretic Preliminaries

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## Definition (Base)

Let  $\alpha \in \mathcal{T}_\omega Q$ , define  $Base(\alpha)$  to be smallest finite set  $X$  such that  $\alpha \in \mathcal{T}_\omega X$ .

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# Preliminaries: Moss' Modality

## Definition (Semantics)

Let  $\mathbb{S} = \langle S, \sigma : S \rightarrow \mathcal{T}S, s_I \rangle$  and  $s \in S$ , then

$$\mathbb{S}, s \Vdash \nabla \alpha \text{ iff } (\sigma(s), \alpha) \in \overline{\mathcal{T}}(\Vdash)$$

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## Example

If  $\mathcal{T} = \mathcal{P}$ , then  $\nabla \alpha \equiv \Box \bigvee \alpha \wedge \bigwedge \diamond[\alpha]$ .

# Preliminaries: Coalgebraic Logic

- ▶  $Q$  a set

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## Definition (Coalgebraic Logic)

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## Definition

- ▶  $\mathcal{L}Q$  is the set of *depth-zero formulas*
- ▶  $\mathcal{T}_\omega^\nabla : X \mapsto \{\nabla\alpha \mid \alpha \in \mathcal{T}_\omega X\}$ .
- ▶  $\mathcal{L}\mathcal{T}_\omega^\nabla\mathcal{L}Q$  is the set of *depth-one formulas*

# One Step Complementation Lemma

## Definition (Boolean Dual of $\nabla$ )

- ▶ Let  $\alpha \in \mathcal{T}_\omega Q$ , define  $D(\alpha) \subseteq \mathcal{T}_\omega \mathcal{P}Q$  as follows

$$D(\alpha) := \left\{ \beta \in \mathcal{T}_\omega \mathcal{P}_\omega \text{Base}(\alpha) \mid \begin{array}{l} \text{for any } R \subseteq \mathcal{P}Q \times Q. (\beta, \alpha) \in \overline{T}R \Rightarrow \\ \text{there is } (b, a) \in R \text{ such that } b \leq a \end{array} \right\}$$

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- ▶ Define  $\Delta\alpha$  as follows

$$\Delta\alpha := \bigvee \left\{ \nabla(T \wedge) \Phi \mid \Phi \in D(\alpha) \right\}.$$

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- ▶ Define  $\Delta\alpha$  as follows

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## Theorem (One-Step Complementation Lemma)

*For all  $\alpha \in \mathcal{T}_\omega Q$ ,  $\nabla\alpha$  and  $\Delta\alpha$  are Boolean duals.*

# One Step Complementation

## Definition (One-Step Dualisation)

$$\delta_0 : \mathcal{L}Q \rightarrow \mathcal{L}Q$$

$$\delta_0(q) := q$$

$$\delta_0(\wedge \phi) := \vee \delta_0[\phi]$$

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$$\delta_1 : \mathcal{L}T_\omega^\nabla \mathcal{L}Q \rightarrow \mathcal{L}T_\omega^\nabla \mathcal{L}Q$$

$$\delta_1(\nabla \alpha) := \Delta(\mathcal{T} \delta_0) \alpha$$

$$\delta_1(\wedge \phi) := \vee \delta_1[\phi]$$

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# One Step Complementation

## Definition (One-Step Dualisation)

$$\begin{array}{ll} \delta_0 : \mathcal{L}Q \rightarrow \mathcal{L}Q & \delta_1 : \mathcal{L}T_\omega^\nabla \mathcal{L}Q \rightarrow \mathcal{L}T_\omega^\nabla \mathcal{L}Q \\ \delta_0(q) & := q & \delta_1(\nabla\alpha) & := \Delta(\mathcal{T}\delta_0)\alpha \\ \delta_0(\wedge\phi) & := \vee\delta_0[\phi] & \delta_1(\wedge\phi) & := \vee\delta_1[\phi] \\ \delta_0(\vee\phi) & := \wedge\delta_0[\phi] & \delta_1(\vee\phi) & := \wedge\delta_1[\phi] \end{array}$$

## Corollary

*For any  $a \in \mathcal{L}T_\omega^\nabla \mathcal{L}Q$ , the depth-one formulas  $a$  and  $\delta_1(a)$  are Boolean duals.*

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# Preliminaries: Parity Graph Games

## Definition (Arena)

Arenas of parity graph games are structures

$$\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega : V \rightarrow \mathbb{N} \rangle$$

- ▶ sets  $V = V_0 \uplus V_1$  of positions
- ▶ an edge relation  $E \subseteq V \times V$
- ▶ an initial position  $v_I \in V$
- ▶ a priority function  $\Omega : V \rightarrow \mathbb{N}$

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## Definition (Winning Condition)

Player  $\Pi \in \{0, 1\}$  (  $\Sigma = 1 - \Pi$  ) wins a play  $p$  of  $\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega \rangle$  if

- ▶  $p$  finite:  $\Sigma$  gets stuck
- ▶  $p$  infinite: largest priority occurring infinitely often has parity  $\Pi$

# Basic Sets

Let  $\mathcal{G}$  be a parity graph game

$$\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega : V \rightarrow \mathbb{N} \rangle$$

## Definition

We call a set  $B \subseteq V$  *basic* if

1.  $v_I \in B$
2. any full play starting at some  $b \in B$  either ends in a terminal position or it passes through another position in  $B$
3.  $\Omega(v) = 0$  iff  $v \notin B$ .

# Local Games

- ▶  $\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega \rangle$  with basic set  $B \subseteq V$ ,  $b \in B$

## Definition (Local Game Trees)

$$\mathcal{T}^b = \langle V_0^b, V_1^b, E^b, (b) \rangle$$

- ▶  $V^b := \{\beta \in V^* \mid \text{first}(\beta) = b, \text{last}(\beta) \in B \Rightarrow \beta = (b)\}$
- ▶  $V_{\Pi}^b := \{\beta \in V^b \mid \text{last}(\beta) \in V_{\Pi}\}$
- ▶  $E^b(\beta) := \{\beta.(v) \mid v \in E(\beta)\}$

# Powers

- ▶  $\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega' \rangle$  with basic set  $B \subseteq V$
- ▶  $b \in B$ ,  $T^b = \langle V_0^b, V_1^b, E^b, (b) \rangle$
- ▶  $\Pi \in \{0, 1\}$ ,  $\Sigma = 1 - \Pi$

## Definition (Powers)

Define the power  $P_\Pi(b) \subseteq B$  of  $\Pi$  at  $b \in B$

- ▶ If  $\beta \in \text{Leaves}(T^b)$ , we put, for each player,

$$P_\Pi(\beta) := \left\{ \{ \text{last}(\beta) \} \right\}.$$

- ▶ If  $\beta \notin \text{Leaves}(T^b)$ , we put

$$P_\Pi(\beta) := \begin{cases} \bigcup \{ P_\Pi(\gamma) \mid \gamma \in E^b(\beta) \} & \text{if } \beta \in V_\Pi^b, \\ \left\{ \bigcup_{\gamma \in E^b(\beta)} Y_\gamma \mid Y_\gamma \in P_\Pi(\gamma), \text{ all } \gamma \right\} & \text{if } \beta \in V_\Sigma^b. \end{cases}$$

- ▶  $P_\Pi(b) := P_\Pi(\langle b \rangle)$

# Powers

- ▶  $\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega \rangle$ , basic set  $B \subseteq V$
- ▶  $\Pi \in \{0, 1\}$ ,  $\Sigma = 1 - \Pi$

## Proposition

*Let  $W$  be a subset of  $B$ . Then the following are equivalent:*

1.  $W \in P_\Pi(b)$ ;
2.  $\Pi$  has a surviving strategy  $f$  in  $\mathcal{G}^b$  such that  $W$  is the set of next basic positions in some play consistent with  $f$

## Proposition

*The following are equivalent*

1.  $\emptyset \in P_\Pi(b)$
2.  $P_\Sigma(b) = \emptyset$
3.  $\Pi$  has a local winning strategy in  $\mathcal{G}^b$

# Game Bisimulation

## Definition (Game Simulation)

- ▶  $\mathcal{G} = \langle V_0, V_1, E, \Omega \rangle$ , basic set  $B \subseteq V$ ,  $\Pi \in \{0, 1\}$
- ▶  $\mathcal{G}' = \langle V'_0, V'_1, E', \Omega' \rangle$ , basic set  $B' \subseteq V'$ ,  $\Pi' \in \{0', 1'\}$

A  $\Pi, \Pi'$ -game simulation is  $Z \subseteq B \times B'$  such that for all  $v \in V$  and  $v' \in V'$  with  $vZv'$ ,  $Z$  satisfies the **structural conditions**

- ▶ (proponent)  $\forall W \in P_{\Pi}^{\mathcal{G}}(v). \exists W' \in P_{\Pi'}^{\mathcal{G}'}(v'). \forall w' \in W'. \exists w \in W. wZw'$ ,
- ▶ (opponent)  $\forall W' \in P_{\Sigma'}^{\mathcal{G}'}(v'). \exists W \in P_{\Sigma}^{\mathcal{G}}(v). \forall w \in W. \exists w' \in W'. wZw'$ ,

and the **priority conditions**

- ▶ (parity)  $\Omega(v) \bmod 2 = \Pi$  iff  $\Omega'(v') \bmod 2 = \Pi'$ ,
- ▶ (contraction) for all  $v, w \in V$  and  $v', w' \in V'$  with  $vZv'$  and  $wZw'$ ,  $\Omega(v) \leq \Omega(w)$  iff  $\Omega(v') \leq \Omega(w')$ .

# Game Bisimulation

## Definition (Game Bisimulation)

- ▶  $\mathcal{G} = \langle V_0, V_1, E, \Omega \rangle$ , basic set  $B \subseteq V$ ,  $\Pi \in \{0, 1\}$
- ▶  $\mathcal{G}' = \langle V'_0, V'_1, E', \Omega' \rangle$ , basic set  $B' \subseteq V'$ ,  $\Pi' \in \{0', 1'\}$

$Z \subseteq B \times B'$  is a  $\Pi, \Pi'$ -game bisimulation if

- ▶  $Z$  is a  $\Pi, \Pi'$ -game simulation
- ▶  $Z^\sim$  is a  $\Pi', \Pi$ -game simulation

## Theorem

*If  $Z \subseteq B \times B'$   $\Pi, \Pi'$ -game bisimulation between parity graph games  $\mathcal{G}$  and  $\mathcal{G}'$ , then*

$$\text{if } vZv' \text{ then } v \in \text{Win}_\Pi(\mathcal{G}) \iff v' \in \text{Win}_{\Pi'}(\mathcal{G}')$$

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# Preliminaries: $\mathcal{T}$ -Automata

## Definition ( $\mathcal{T}$ -Automata in Logical Form)

Alternating  $\mathcal{T}$ -automata are structures

$$\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}Q, q_I, \Omega \rangle$$

consisting of

- ▶ a *finite* set  $Q$  of states
- ▶ a transition function  $\theta : Q \rightarrow \mathcal{L}\mathcal{T}Q$
- ▶ an initial state  $q_I \in Q$
- ▶ a priority function  $\Omega : Q \rightarrow \mathbb{N}$

# Preliminaries: $\mathcal{T}$ -Automata

- ▶  $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}Q, q_I, \Omega \rangle$  an alternating automaton
- ▶  $\mathbb{S} = \langle S, \sigma : S \rightarrow \mathcal{T}S, s_I \rangle$  a pointed  $\mathcal{T}$ -coalgebra

## Definition

Acceptance games are parity graph games

$$\mathcal{G}(\mathbb{A}, \mathbb{S}) = \langle V_{\exists}, V_{\forall}, E, (q_I, s_I), \Omega_{\mathcal{G}} \rangle$$

Position		Sets of Admissible Moves	$\Omega_{\mathcal{G}}$
$(q, s) \in Q \times S$	-	$\{(\theta(q), s)\}$	$\Omega(q)$
$(\bigwedge \tau, s) \in \mathcal{L}\mathcal{T}^{\nabla}Q \times S$	$\forall$	$\{(q, s) \mid q \in \tau\}$	0
$(\bigvee \tau, s) \in \mathcal{L}\mathcal{T}^{\nabla}Q \times S$	$\exists$	$\{(q, s) \mid q \in \tau\}$	0
$(\nabla \alpha, s) \in \mathcal{T}^{\nabla}Q \times S$	$\exists$	$\{Z \subseteq Q \times S \mid (\alpha, \sigma(s)) \in \overline{\mathcal{T}Z}\}$	0
$Z \subseteq Q \times S$	$\forall$	$Z$	0

# Bird-eye-view on $\mathcal{T}$ -Automata

- ▶  $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}Q, q_I, \Omega : Q \rightarrow \mathbb{N} \rangle$

Transition structure

- ▶  $\theta$  is  $\mathcal{L}$ -structured  $\mathcal{T}$ -coalgebra pointed in  $q_I$

Semantics

- ▶ finitary and infinitary trace semantics
- ▶ parameterised in  $\Omega$

# Transalternating Automata

- ▶ Alternating  $\mathcal{T}$ -Aut'a:  $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla Q, q_I, \Omega \rangle$

## Definition (Transalternating Automata)

$$\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla \mathcal{L}Q, q_I, \Omega \rangle$$

## Definition (Acceptance Games)

*similar to acceptance games for alternating  $\mathcal{T}$ -automata*

# Complements of Transalternating Automata

- ▶  $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{LTL}Q, q_I, \Omega \rangle$  a transalternating  $\mathcal{T}$ -automaton

## Definition (Complements of Transalternating Automata)

Define the complementary automaton

$$\mathbb{A}^c = \langle Q, \theta^c : Q \rightarrow \mathcal{LTL}Q, q_I, \Omega^c \rangle$$

such that

- ▶  $\theta^c(q) := \delta_1(\theta(q))$
- ▶  $\Omega^c(q) := \Omega(q) + 1$ , for all  $q \in Q$ .

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- ▶  $\Omega^c(q) := \Omega(q) + 1$ , for all  $q \in Q$ .

## Theorem

*For every transalternating  $\mathcal{T}$ -coalgebra automaton  $\mathbb{A}$ , the automaton  $\mathbb{A}^c$  accepts precisely those pointed  $\mathcal{T}$ -coalgebras that are rejected by  $\mathbb{A}$ .*

# Transalternating and Alternating Automata

- ▶ Alternating  $\mathcal{T}$ -Aut'a:  $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla Q, q_I, \Omega \rangle$
- ▶ Transalternating  $\mathcal{T}$ -Aut'a:  $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla \mathcal{L}Q, q_I, \Omega \rangle$

## Definition (Semi-Transalternating Automata)

$$\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla \mathcal{S}Q, q_I, \Omega \rangle$$

# Transalternating and Alternating Automata

- ▶ Alternating  $\mathcal{T}$ -Aut'a:  $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla Q, q_I, \Omega \rangle$
- ▶ Transalternating  $\mathcal{T}$ -Aut'a:  $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla \mathcal{L}Q, q_I, \Omega \rangle$

## Definition (Semi-Transalternating Automata)

$$\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla \mathcal{S}Q, q_I, \Omega \rangle$$

## Theorem

*There is an effective translation between*

1. *Alternating Automata*
2. *Transalternating Automata*
3. *Semi-Transalternating Automata*

*We showed  $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$*

# Size Matters

## Theorem

*For every alternating automaton  $\mathbb{A}$  with  $n$  states **there is an alternating automaton  $\mathbb{A}^c$  complementary to  $\mathbb{A}$  with  $2^n \times n$  states.***

# Size Matters

## Theorem

For every alternating automaton  $\mathbb{A}$  with  $n$  states **there is an alternating automaton  $\mathbb{A}^c$  complementary to  $\mathbb{A}$  with  $2^n \times n$  states.**

## Theorem

If  $\mathcal{T}$  is such that for any  $\nabla\alpha \in \mathcal{T}^\nabla Q$ ,  $\Delta\alpha \in \mathcal{L}\mathcal{T}^\nabla Q$ , then for any alternating  $\mathcal{T}$ -automaton of  $n$  states **there is a complementing alternating automaton with at most  $n + c$  states, for some constant  $c$ .**

# Some Conclusions

## Summary

- ▶ Effective Complementation for Coalgebra Automata
- ▶ Coinductive Method of Game (Bi)Simulation for (some) Parity Graph Games

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## Corollaries

- ▶ (Boolean) Coalgebraic Logic is Negation-free
- ▶ Correspondence between Second-Order Coalgebraic Logic and Coalgebra Automata

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## Open Questions

- ▶ Categorical Nature of the Correspondence
- ▶ Moss' Coalgebraic Logic and Coalgebraic Modal Logic and Fixed-Point Operators
- ▶ Characterisation of Game (Bi)Similarity

# Conclusions and References

**Thank You**, and the authors of

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