

Complementation of Coalgebra Automata

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Goal of the Talk

Theorem

The class of languages recognisable by \mathcal{T} -coalgebra automata is closed under taking complements.

Outline

1. One Step Complementation Lemma
 - ▶ Moss' Modality
 - ▶ Boolean Dual of Moss' Modality
2. Game Bisimulation
 - ▶ Parity Graph Games
 - ▶ Basic Sets and Local Games
 - ▶ Powers and Game Normalisation
 - ▶ Game Bisimulation
3. Complementation Lemma for Coalgebra Automata
 - ▶ Coalgebra Automata
 - ▶ Complementation of Transalternating Automata
 - ▶ Equivalence of Transalternating, Semi-transalternating and Alternating Automata

Category-Theoretic Preliminaries

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Definition (Base)

Let $\alpha \in \mathcal{T}_\omega Q$, define $Base(\alpha)$ to be smallest finite set X such that $\alpha \in \mathcal{T}_\omega X$.

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Preliminaries: Moss' Modality

Definition (Semantics)

Let $\mathbb{S} = \langle S, \sigma : S \rightarrow \mathcal{T}S, s_I \rangle$ and $s \in S$, then

$$\mathbb{S}, s \Vdash \nabla \alpha \text{ iff } (\sigma(s), \alpha) \in \overline{\mathcal{T}}(\Vdash)$$

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Example

If $\mathcal{T} = \mathcal{P}$, then $\nabla \alpha \equiv \Box \bigvee \alpha \wedge \bigwedge \diamond[\alpha]$.

Preliminaries: Coalgebraic Logic

- ▶ Q a set

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Definition (Coalgebraic Logic)

A logic is called a coalgebraic logic, if it contains ∇ as a connective.

Definition

- ▶ $\mathcal{L}Q$ is the set of *depth-zero formulas*
- ▶ $\mathcal{T}_\omega^\nabla : X \mapsto \{\nabla\alpha \mid \alpha \in \mathcal{T}_\omega X\}$.
- ▶ $\mathcal{L}\mathcal{T}_\omega^\nabla\mathcal{L}Q$ is the set of *depth-one formulas*

One Step Complementation Lemma

Definition (Boolean Dual of ∇)

- ▶ Let $\alpha \in \mathcal{T}_\omega Q$, define $D(\alpha) \subseteq \mathcal{T}_\omega \mathcal{P}Q$ as follows

$$D(\alpha) := \left\{ \begin{array}{l} \beta \in \mathcal{T}_\omega \mathcal{P}_\omega \text{Base}(\alpha) \mid \\ \text{for any } R \subseteq \mathcal{P}Q \times Q. (\beta, \alpha) \in \overline{T}R \Rightarrow \\ \text{there is } (b, a) \in R \text{ such that } b \leq a \end{array} \right\}$$

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- ▶ Define $\Delta\alpha$ as follows

$$\Delta\alpha := \bigvee \left\{ \nabla(T \wedge) \Phi \mid \Phi \in D(\alpha) \right\}.$$

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- ▶ Define $\Delta\alpha$ as follows

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Theorem (One-Step Complementation Lemma)

For all $\alpha \in \mathcal{T}_\omega Q$, $\nabla\alpha$ and $\Delta\alpha$ are Boolean duals.

One Step Complementation

Definition (One-Step Dualisation)

$$\delta_0 : \mathcal{L}Q \rightarrow \mathcal{L}Q$$

$$\delta_0(q) := q$$

$$\delta_0(\wedge \phi) := \vee \delta_0[\phi]$$

$$\delta_0(\vee \phi) := \wedge \delta_0[\phi]$$

$$\delta_1 : \mathcal{L}T_\omega^\nabla \mathcal{L}Q \rightarrow \mathcal{L}T_\omega^\nabla \mathcal{L}Q$$

$$\delta_1(\nabla \alpha) := \Delta(\mathcal{T} \delta_0) \alpha$$

$$\delta_1(\wedge \phi) := \vee \delta_1[\phi]$$

$$\delta_1(\vee \phi) := \wedge \delta_1[\phi]$$

One Step Complementation

Definition (One-Step Dualisation)

$$\begin{array}{ll} \delta_0 : \mathcal{L}Q \rightarrow \mathcal{L}Q & \delta_1 : \mathcal{L}T_\omega^\nabla \mathcal{L}Q \rightarrow \mathcal{L}T_\omega^\nabla \mathcal{L}Q \\ \delta_0(q) & := q & \delta_1(\nabla\alpha) & := \Delta(\mathcal{T}\delta_0)\alpha \\ \delta_0(\wedge\phi) & := \vee\delta_0[\phi] & \delta_1(\wedge\phi) & := \vee\delta_1[\phi] \\ \delta_0(\vee\phi) & := \wedge\delta_0[\phi] & \delta_1(\vee\phi) & := \wedge\delta_1[\phi] \end{array}$$

Corollary

For any $a \in \mathcal{L}T_\omega^\nabla \mathcal{L}Q$, the depth-one formulas a and $\delta_1(a)$ are Boolean duals.

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 - ▶ **Powers and Game Normalisation**
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Preliminaries: Parity Graph Games

Definition (Arena)

Arenas of parity graph games are structures

$$\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega : V \rightarrow \mathbb{N} \rangle$$

- ▶ sets $V = V_0 \uplus V_1$ of positions
- ▶ an edge relation $E \subseteq V \times V$
- ▶ an initial position $v_I \in V$
- ▶ a priority function $\Omega : V \rightarrow \mathbb{N}$

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Definition (Winning Condition)

Player $\Pi \in \{0, 1\}$ ($\Sigma = 1 - \Pi$) wins a play p of $\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega \rangle$ if

- ▶ p finite: Σ gets stuck
- ▶ p infinite: largest priority occurring infinitely often has parity Π

Basic Sets

Let \mathcal{G} be a parity graph game

$$\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega : V \rightarrow \mathbb{N} \rangle$$

Definition

We call a set $B \subseteq V$ *basic* if

1. $v_I \in B$
2. any full play starting at some $b \in B$ either ends in a terminal position or it passes through another position in B
3. $\Omega(v) = 0$ iff $v \notin B$.

Local Games

- ▶ $\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega \rangle$ with basic set $B \subseteq V$, $b \in B$

Definition (Local Game Trees)

$$\mathcal{T}^b = \langle V_0^b, V_1^b, E^b, (b) \rangle$$

- ▶ $V^b := \{\beta \in V^* \mid \text{first}(\beta) = b, \text{last}(\beta) \in B \Rightarrow \beta = (b)\}$
- ▶ $V_{\Pi}^b := \{\beta \in V^b \mid \text{last}(\beta) \in V_{\Pi}\}$
- ▶ $E^b(\beta) := \{\beta.(v) \mid v \in E(\beta)\}$

Powers

- ▶ $\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega' \rangle$ with basic set $B \subseteq V$
- ▶ $b \in B$, $T^b = \langle V_0^b, V_1^b, E^b, (b) \rangle$
- ▶ $\Pi \in \{0, 1\}$, $\Sigma = 1 - \Pi$

Definition (Powers)

Define the power $P_\Pi(b) \subseteq B$ of Π at $b \in B$

- ▶ If $\beta \in \text{Leaves}(T^b)$, we put, for each player,

$$P_\Pi(\beta) := \left\{ \{ \text{last}(\beta) \} \right\}.$$

- ▶ If $\beta \notin \text{Leaves}(T^b)$, we put

$$P_\Pi(\beta) := \begin{cases} \bigcup \{ P_\Pi(\gamma) \mid \gamma \in E^b(\beta) \} & \text{if } \beta \in V_\Pi^b, \\ \left\{ \bigcup_{\gamma \in E^b(\beta)} Y_\gamma \mid Y_\gamma \in P_\Pi(\gamma), \text{ all } \gamma \right\} & \text{if } \beta \in V_\Sigma^b. \end{cases}$$

- ▶ $P_\Pi(b) := P_\Pi(\langle b \rangle)$

Powers

- ▶ $\mathcal{G} = \langle V_0, V_1, E, v_I, \Omega \rangle$, basic set $B \subseteq V$
- ▶ $\Pi \in \{0, 1\}$, $\Sigma = 1 - \Pi$

Proposition

Let W be a subset of B . Then the following are equivalent:

1. $W \in P_\Pi(b)$;
2. Π has a surviving strategy f in \mathcal{G}^b such that W is the set of next basic positions in some play consistent with f

Proposition

The following are equivalent

1. $\emptyset \in P_\Pi(b)$
2. $P_\Sigma(b) = \emptyset$
3. Π has a local winning strategy in \mathcal{G}^b

Game Bisimulation

Definition (Game Simulation)

- ▶ $\mathcal{G} = \langle V_0, V_1, E, \Omega \rangle$, basic set $B \subseteq V$, $\Pi \in \{0, 1\}$
- ▶ $\mathcal{G}' = \langle V'_0, V'_1, E', \Omega' \rangle$, basic set $B' \subseteq V'$, $\Pi' \in \{0', 1'\}$

A Π, Π' -game simulation is $Z \subseteq B \times B'$ such that for all $v \in V$ and $v' \in V'$ with vZv' , Z satisfies the **structural conditions**

- ▶ (proponent) $\forall W \in P_{\Pi}^{\mathcal{G}}(v). \exists W' \in P_{\Pi'}^{\mathcal{G}'}(v'). \forall w' \in W'. \exists w \in W. wZw'$,
- ▶ (opponent) $\forall W' \in P_{\Sigma'}^{\mathcal{G}'}(v'). \exists W \in P_{\Sigma}^{\mathcal{G}}(v). \forall w \in W. \exists w' \in W'. wZw'$,

and the **priority conditions**

- ▶ (parity) $\Omega(v) \bmod 2 = \Pi$ iff $\Omega'(v') \bmod 2 = \Pi'$,
- ▶ (contraction) for all $v, w \in V$ and $v', w' \in V'$ with vZv' and wZw' , $\Omega(v) \leq \Omega(w)$ iff $\Omega(v') \leq \Omega(w')$.

Game Bisimulation

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- ▶ $\mathcal{G}' = \langle V'_0, V'_1, E', \Omega' \rangle$, basic set $B' \subseteq V'$, $\Pi' \in \{0', 1'\}$

$Z \subseteq B \times B'$ is a Π, Π' -game bisimulation if

- ▶ Z is a Π, Π' -game simulation
- ▶ Z^\sim is a Π', Π -game simulation

Theorem

If $Z \subseteq B \times B'$ Π, Π' -game bisimulation between parity graph games \mathcal{G} and \mathcal{G}' , then

$$\text{if } vZv' \text{ then } v \in \text{Win}_\Pi(\mathcal{G}) \iff v' \in \text{Win}_{\Pi'}(\mathcal{G}')$$

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Preliminaries: \mathcal{T} -Automata

Definition (\mathcal{T} -Automata in Logical Form)

Alternating \mathcal{T} -automata are structures

$$\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}Q, q_I, \Omega \rangle$$

consisting of

- ▶ a *finite* set Q of states
- ▶ a transition function $\theta : Q \rightarrow \mathcal{L}\mathcal{T}Q$
- ▶ an initial state $q_I \in Q$
- ▶ a priority function $\Omega : Q \rightarrow \mathbb{N}$

Preliminaries: \mathcal{T} -Automata

- ▶ $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}Q, q_I, \Omega \rangle$ an alternating automaton
- ▶ $\mathbb{S} = \langle S, \sigma : S \rightarrow \mathcal{T}S, s_I \rangle$ a pointed \mathcal{T} -coalgebra

Definition

Acceptance games are parity graph games

$$\mathcal{G}(\mathbb{A}, \mathbb{S}) = \langle V_{\exists}, V_{\forall}, E, (q_I, s_I), \Omega_G \rangle$$

Position		Sets of Admissible Moves	Ω_G
$(q, s) \in Q \times S$	-	$\{(\theta(q), s)\}$	$\Omega(q)$
$(\bigwedge \tau, s) \in \mathcal{L}\mathcal{T}^{\nabla}Q \times S$	\forall	$\{(q, s) \mid q \in \tau\}$	0
$(\bigvee \tau, s) \in \mathcal{L}\mathcal{T}^{\nabla}Q \times S$	\exists	$\{(q, s) \mid q \in \tau\}$	0
$(\nabla \alpha, s) \in \mathcal{T}^{\nabla}Q \times S$	\exists	$\{Z \subseteq Q \times S \mid (\alpha, \sigma(s)) \in \overline{\mathcal{T}Z}\}$	0
$Z \subseteq Q \times S$	\forall	Z	0

Bird-eye-view on \mathcal{T} -Automata

- ▶ $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}Q, q_I, \Omega : Q \rightarrow \mathbb{N} \rangle$

Transition structure

- ▶ θ is \mathcal{L} -structured \mathcal{T} -coalgebra pointed in q_I

Semantics

- ▶ finitary and infinitary trace semantics
- ▶ parameterised in Ω

Transalternating Automata

- ▶ Alternating \mathcal{T} -Aut'a: $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla Q, q_I, \Omega \rangle$

Definition (Transalternating Automata)

$$\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla \mathcal{L}Q, q_I, \Omega \rangle$$

Definition (Acceptance Games)

similar to acceptance games for alternating \mathcal{T} -automata

Complements of Transalternating Automata

- ▶ $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{LTL}Q, q_I, \Omega \rangle$ a transalternating \mathcal{T} -automaton

Definition (Complements of Transalternating Automata)

Define the complementary automaton

$$\mathbb{A}^c = \langle Q, \theta^c : Q \rightarrow \mathcal{LTL}Q, q_I, \Omega^c \rangle$$

such that

- ▶ $\theta^c(q) := \delta_1(\theta(q))$
- ▶ $\Omega^c(q) := \Omega(q) + 1$, for all $q \in Q$.

Complements of Transalternating Automata

- ▶ $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{LTL}Q, q_I, \Omega \rangle$ a transalternating \mathcal{T} -automaton

Definition (Complements of Transalternating Automata)

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Theorem

For every transalternating \mathcal{T} -coalgebra automaton \mathbb{A} , the automaton \mathbb{A}^c accepts precisely those pointed \mathcal{T} -coalgebras that are rejected by \mathbb{A} .

Transalternating and Alternating Automata

- ▶ Alternating \mathcal{T} -Aut'a: $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla Q, q_I, \Omega \rangle$
- ▶ Transalternating \mathcal{T} -Aut'a: $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla \mathcal{L}Q, q_I, \Omega \rangle$

Definition (Semi-Transalternating Automata)

$$\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla \mathcal{S}Q, q_I, \Omega \rangle$$

Transalternating and Alternating Automata

- ▶ Alternating \mathcal{T} -Aut'a: $\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla Q, q_I, \Omega \rangle$
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Definition (Semi-Transalternating Automata)

$$\mathbb{A} = \langle Q, \theta : Q \rightarrow \mathcal{L}\mathcal{T}^\nabla \mathcal{S}Q, q_I, \Omega \rangle$$

Theorem

There is an effective translation between

1. *Alternating Automata*
2. *Transalternating Automata*
3. *Semi-Transalternating Automata*

We showed $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$

Size Matters

Theorem

*For every alternating automaton \mathbb{A} with n states **there is an alternating automaton \mathbb{A}^c complementary to \mathbb{A} with $2^n \times n$ states.***

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For every alternating automaton \mathbb{A} with n states **there is an alternating automaton \mathbb{A}^c complementary to \mathbb{A} with $2^n \times n$ states.**

Theorem

If \mathcal{T} is such that for any $\nabla\alpha \in \mathcal{T}^\nabla Q$, $\Delta\alpha \in \mathcal{L}\mathcal{T}^\nabla Q$, then for any alternating \mathcal{T} -automaton of n states **there is a complementing alternating automaton with at most $n + c$ states, for some constant c .**

Some Conclusions

Summary

- ▶ Effective Complementation for Coalgebra Automata
- ▶ Coinductive Method of Game (Bi)Simulation for (some) Parity Graph Games

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Corollaries

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- ▶ Correspondence between Second-Order Coalgebraic Logic and Coalgebra Automata

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Open Questions

- ▶ Categorical Nature of the Correspondence
- ▶ Moss' Coalgebraic Logic and Coalgebraic Modal Logic and Fixed-Point Operators
- ▶ Characterisation of Game (Bi)Similarity

Conclusions and References

Thank You, and the authors of

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